CONVECTIVE DIFFUSION TO SOLID SPHERICAL PARTICLES IN A DENSE POLYDISPERSE CLOUD

Yu. A. Buevich

UDC 532.73-3

Expressions are derived for the diffusion current and the corresponding coefficient of mass transfer to solid spherical particles in a constricted stream of fluid with a low Reynolds number and a high Peclet number.

When simultaneously the Reynolds number is low $Re = 2aU/\nu < 1$ and the Peclet number is high $Pe = 2aU/D \gg 1$, then the stream around a particle can be treated in the Stokes approximation and the diffusive heat or mass transfer to its surface can be analyzed in terms of a diffusion boundary layer. Many studies have been published with a solution to the equation of convective diffusion to a particle where $Re < 1$ and $Pe \gg 1$. The gist of the methods used in those studies, however, was either a transformation of this equation into the equation of plain diffusion as proposed by Levich [1] or a modification of the Karman–Polhausen polynomial according to Aksel'rud for the diffusion boundary layer [2].

Extending these methods to cases with a high volume concentration of particles leads to difficulties in determining the flow field around individual particles. In studies on this subject [3-6] one has used the characteristics of constricted flow, on the basis of various semiempirical cellular models describing the flow of a fluid through a dense cloud of particles. In this article the problem of diffusion to a particle will be solved on the basis of more rigorous stipulations concerning a constricted flow, arrived at by the method shown in [7-8].

The equation of convective diffusion, with axial symmetry of the process taken into account, is in spherical coordinates (the critical point at the particle surface has the coordinate $\theta = 0$)

$$v_r \frac{\partial c}{\partial r} + \frac{v_\theta}{r} \frac{\partial c}{\partial \theta} = \frac{D}{r^2} \left[\frac{\partial}{\partial r} \left(r^2 \frac{\partial c}{\partial r} \right) + \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial c}{\partial \theta} \right) \right], \tag{1}$$

with v_r and v_θ denoting the velocity components. Assuming, for simplicity, that mass is absorbed at the particle surface at a high rate, we can write the boundary conditions for Eq. (1) as

$$c = c_0 \ (r \to \infty;\ r = a,\ \theta = 0), \quad c = 0 \ (r = a,\ \theta \neq 0), \tag{2}$$

with c_0 denoting the concentration of the substance in the oncoming stream.

If one considers that the diffusion boundary layer is thin (its thickness is $h(\theta) \ll a$ everywhere except, perhaps, in the vicinity of the stern point $\theta = \pi$), the tangential derivatives within this layer are negligible in comparison with the radial derivatives and Eq. (1) becomes

$$v_r \frac{\partial c}{\partial \xi} + \frac{v_\theta}{a} \frac{\partial c}{\partial \theta} \approx D \frac{\partial^2 c}{\partial \xi^2}, \quad r = a + \xi, \quad 0 \leqslant \xi \leqslant h(\theta) \ll a. \tag{3}$$

Using the results in [7], in order to make the problem determinate, we express the velocity v_θ and the flow function ψ near the surface of a particle in the approximate form

$$v_\theta \approx \frac{3}{2} \frac{\xi}{a} U_* \sin \theta, \quad \psi \approx -\frac{3}{4} U_* \xi^2 \sin^2 \theta, \quad U_* = AU, \tag{4}$$

Institute of Problems in Mechanics, Academy of Sciences of the USSR, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 23, No. 4, pp. 709-712, October, 1972. Original article submitted February 4, 1972.

where A is a function of the volume concentration ρ and of the cloud dispersivity

$$A = (1 + \beta(\rho, \varphi)), \quad \beta = \frac{a}{2 - 3\rho}\left\{\left[18\rho\left(1 - \frac{3}{2}\rho\right)\frac{b_1}{b_3} + \frac{81}{4}\rho^2\left(\frac{b_2}{b_3}\right)^2\right]^{1/2} + \frac{9}{2}\rho\frac{b_2}{b_3}\right\},$$

$$\rho = \frac{4}{3}\pi n \int a^3\varphi(a)\,da, \quad b_m = \int a^m\varphi(a)\,da, \tag{5}$$

with n denoting the numerical concentration of particles and $\varphi(a)$ the normal-size distribution function of particles.

When $\rho \to 0$, Eq. (5) yields $\beta \sim \sqrt{\rho}$. In the special case of a monodisperse cloud $b_m = a^m$ and

$$\beta = \frac{1}{2 - 3\rho}\left\{\left[18\rho\left(1 - \frac{3}{2}\rho\right) + \frac{81}{4}\rho^2\right]^{1/2} + \frac{9}{2}\rho\right\}, \tag{6}$$

With the aid of (4) it is not difficult to replace r (or ξ) by a new independent variable ψ and to transform the boundary conditions (2) as well as Eq. (3) accordingly. The solution to the resulting boundary-value problem does not differ in any way from the solution to the single-particle problem in [1], if the relative velocity U is replaced by U_* from expression (4). As a result, we have for the local and the integral diffusion current at the surface of a particle

$$j(\theta) = D\left(\frac{\partial c}{\partial \xi}\right)_{\xi=0} = 0.79\left(\frac{AUD^2}{a^2}\right)^{1/3} c_0 \frac{\sin\theta}{(\theta - 1/2\sin 2\theta)^{1/3}},$$

and

$$J = 2\pi a^2 \int_0^\pi j(\theta)\sin\theta d\theta = 7.98\,(D^2AUa^4)^{1/3}, \tag{7}$$

respectively.

The thickness of the diffusion boundary layer is

$$h(\theta) = 1.27\left(\frac{Da^2}{AU}\right)^{1/3}\frac{(\theta - 1/2\sin 2\theta)^{1/3}}{\sin\theta}. \tag{8}$$

As a result of a constricted flow with Re < 1 and Pe ≫ 1, therefore, the diffusion current to a particle becomes $A^{1/3}$ times larger and the thickness of the diffusion boundary layer becomes $A^{1/3}$ times smaller than in the case of a single particle, with A defined according to expressions (5) and (6). When $\rho \to 0$, we have $A \to 1$ and formulas (7)-(8) become the corresponding ones in [1].

Introducing the Sherwood number Sh = $2ak/D$, where k is the integral mass transfer coefficient defined as the ratio of current J to the quantity $4\pi a^2 c_0$, we obtain from (7) the criterial relation

$$S = BP^{1/3}, \quad B = 0.998\,A^{1/3}. \tag{9}$$

The diffusion current J can also be calculated by the polynomial method. Namely, integrating (1) or (3) with respect to ξ from 0 to $h(\theta)$, one can obtain the condition of material balance in the diffusion layer (at $\xi = h(\theta)$ the concentration is $c = c_0$), express the concentration of a substance as a polynomial in terms of the $\xi/h(\theta)$ ratio, and determine the polynomial coefficients so as to satisfy the boundary conditions [2, 6]. This method, the shortcomings of which have been discussed in [1], leads to the earlier derived relation (9) between the Sherwood number and the Peclet number, but coefficient B is here

$$B = 1.037\,A^{1/3}. \tag{10}$$

The relations derived in [3-6] for particles of a monodisperse cloud are of the same form as (9), but there

$$B = K\left(\frac{1 - \rho^{5/3}}{1 + 3/2\,\rho^{5/3} - \rho^{1/3}(3/2 + \rho^{5/3})}\right)^{1/3}, \tag{11}$$

and the numerical coefficient K in (11) is equal to 1.19 according to Ruckenstein [3], 0.998 according to Pfeffer [4] or Walso and Gal-Or [5], and 1.037 according to Yaron and Gal-Or [6].

The results obtained here are valid for fine particles (Re < 1). Their extension to cases with the Reynolds number somewhat higher than unity is fraught with difficulties (in the case of single particles) arising in the description of the flow near a particle in terms of adjoint asymptotic expansions. For particles in a rather dense cloud, however, the results obtained in the Re < 1 approximation should be valid also at higher values of the Reynolds number (Re = 10-100). This has to do with a much softer separation of the boundary layer formed during the flow through a dense cloud of particles than in the case of single particles. This situation has been discussed and illustrated with some test data in [4, 5]. It has also been confirmed by direct observations of the flow through a close-packed cubic lattice of spheres in [9], according to which an effective separation of the boundary layer occurs only at Re = 90-120. It does not seem to be particularly meaningful, therefore, to express the integral diffusion current J through a dense cloud in terms of a series in Re – as is done in the case of single particles.

NOTATION

A is a quantity defined by Eq. (5);
a is the radius of a particle;
B is the coefficient in formula (9);
b_m are the moments of function $\varphi(a)$;
c is the concentration;
D is the molecular diffusivity;
J is the integral diffusion current at the surface of a spherical particle;
j is the local diffusion current;
h is the thickness of the diffusion boundary layer;
K is the coefficient in formula (11);
k is the integral mass transfer coefficient;
Pe is the Peclet number;
Re is the Reynolds number;
Sh is the Sherwood number;
U is the relative velocity;
U_* is the velocity as defined in Eq. (4);
v is the local velocity of the fluid;
β is the coefficient in Eq. (5);
ν is the kinematic viscosity;
ρ is the volume concentration of particles;
φ is the size (radius) distribution function of particles;
ψ is the flow function.

LITERATURE CITED

1. V. G. Levich, Physicochemical Hydrodynamics [in Russian], Fizmatgiz, Moscow (1959).
2. G. A. Aksel'rud, Zh. Fiz. Khim., 27, No. 1 (1953).
3. E. Ruckenstein, Chem. Eng. Sci., 19, 131 (1964).
4. R. Pfeffer, Ind. Eng. Che. Fund., 3, 380 (1964).
5. S. Waslo and B. Gal-Or, Chem. Eng. Sci., 26, 829 (1971).
6. I. Yaron and B. Gal-Or, Internatl. J. Heat and Mass Transfer, 14, 727 (1971).
7. C. K. W. Tam, J. Fluid Mech., 38, 537 (1969).
8. Yu. A. Buevich, Izv. Akad. Nauk SSSR, Mekhanika Zhidkosti i Gaza, No. 5 (1971).
9. T. H. Wegner, A. J. Karabelas, and T. J. Hanratty, Chem. Eng. Sci., 26, 59 (1971).